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(PARTE I).

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"INTERNATIONAL SYMPOSIUM ON ELECTRON AND POSITRON
STORAGE RINGS" - Saclay, September 1966

PARTE I

1) Adone status report

F. Amman, R. Andreani, M. Bassetti, M. Bernardini, A.
Cattoni, V. Chimenti, G. Corazza, D. Fabiani, E. Fer-
lenghi, A. Massarotti, C. Pellegrini, M. Placidi, M. Pu-
glisi, F. Soso, S. Tazzari, F. Tazzioli and A. Tenore.

2) Instability of intense electron beam and radiofrequency
system in a storage ring

M. Bassetti.

3) Increase of the longitudinal dimension of a relativistic
bunch due to the coherent-radiation force

E. Ferlenghi.

4) Remarks on multiple Coulomb scattering in a relativistic
electron beam, beam size and lifetime

C. Pellegrini.

1) - ADONE STATUS REPORT

F. Amman, R. Andreani, M. Bassetti, M. Bernardini, A. Cattoni, V. Chimenti, G. Corazza, D. Fabiani, E. Ferlenghi, A. Massarotti, C. Pellegrini, M. Placidi, M. Puglisi, F. Soso, S. Tazzari, F. Tazzioli, A. Tenore.
(Presented by F. Amman)

Introduzione. -

The storage ring data are collected in table I; details of the construction can be found in references[1,2].

The lay-out is shown in fig. 1; there are four experimental straight sections; depending on the operation mode the beam crossings can take place in one pair at a time (diametrically opposed), or in all four. The straight sections are 2.5 meters long (from quadrupole ends); the room that must be left free for the beam is, in cross section, 22×6 cm^2 (inside aperture). At the beginning of operation all the straight sections will have a circular vacuum chamber, 25 cm in diameter.

The design figure for the luminosity at the maximum energy of 1,5 GeV per beam is 10^{33} cm^{-2} hr^{-1}; it corresponds to a betatron wave number variation $\times$ Q_V of 0,025 per crossing (with $Q_V \simeq 3.1$). The computations[3] show that it should be possible to achieve a $\delta Q_V \simeq 0.1$; the corresponding luminosity should be then 7×10^{33} cm^{-2} hr^{-1}.

Injector. -

The $e^+ e^-$ linear accelerator, built by Varian Ass. (Palo Alto) for CNR (National Research Council) is now under test in Frascati (fig. 2).

Preliminary results[4] show that its performances, for positrons and electrons, are quite close to the calculated values, but for the energy, which is still low; a few sections cannot be operated at full power because of RF windows troubles.

Figg. 3 and 4 show positron and electron peak current spectra; in a 2% energy bin (the ring acceptance) we have 0.58 mA of positrons (with a total accelerated current of about 0.93 mA) and almost 90 mA of electrons (more than 90% of the accelerated current). The positron results have been obtained with an electron beam of 250 mA peak at about 80 MeV.

It is interesting to compare these results with those obtained in Palo Alto at the end of the first high energy section (with the same electron beam on the converter[2]) and with the computations[5,6].

TABLE I

STORAGE RING DATA . -

Particles stored		e^+, e^-
Maximum energy	(GeV)	1.5
Intensity, per beam	(part)	2×10^{11}
Storage time	(hrs)	~ 0.5
Crossing regions free for experiments		4
Luminosity at 1.5 GeV	$(cm^{-2}hrs^{-1})$	10^{33}

Magnet

Focusing, type		AG, separated functions
Focusing, order		$0/2Q_F Q_D B Q_D Q_F 0/2$
Field index, n, in bending magnets		0.5
Field gradient in quadrupoles, at max energy	(Gs/cm)	380
Field, at injection in the magnets	(kGs)	2.4
Field, at max energy in the magnets	(kGs)	10
Orbit radius	(m)	5.00
Mean radius	(m)	16.71
Numbers of periods		12
Betatron wave numbers (variable)		3.1 ± 0.5
Closed orbit amplitude for	(cm)	1.91 max
$(\Delta p/p) = 1\%$ and $Q_R = 3.2$	(cm)	0.96 min
Momentum compaction		6.12×10^{-2}
Damping time constants for inj;	(msec)	800
betatron oscillations max energy	(msec)	11
Iron weight	(ton)	300
Copper weight	(ton)	30
Power, at 1.5 GeV	(kW)	1300
Useful aperture: width	(cm)	22
height	(cm)	10

Injector

Type		S-band linac
Injection energy, e^-	(MeV)	375
e^+	(MeV)	360
Injector current within e^-	(mA)	$\gtrsim 75$
2% energy bin e^+	(mA)	$\gtrsim 0.5$
Injection repetition rate	(pps)	1.5 ÷ 3

Vacuum system

Design pressure	(torr)	10^{-9}
Pumps, type		getter ion
Pumps, number and size		24 x 500 lt/sec

RF system

Frequency	(Mc/sec)	8.58
Harmonic number		3
Accelerating cavities		2 x 2
Max voltage per turn	(kV)	200
Input power to RF cavities, max	(kW)	190
Power delivered to the beam, at 1.5 GeV	(kW)	18

According to the computations the total positron current should have been 1. 1 mA 60% of which in 1% energy bin; we list here the assumptions made in the computations which, in some way, differ from the actual situation:

a) uniform distribution in angles of the positron source; results obtained at Stanford[7] with 1 GeV electrons and $6 \div 14$ MeV positrons out of the converter show a exp $- \theta/\theta_0$ distribution with $\theta_0 \simeq$ $\simeq 0.35$ rad; if we apply this distribution to our computations the results for the total current should be multiplied by $\simeq 0.75$;

b) the integrated electric field increases with radius: at 1 cm is 1% higher than on axis; measurements made by J. Haimson at Varian Ass. on our wave guides give a field rise of 1.6% at 0.75 cm; this should effect only the energy spread, not the total current;

c) the axial magnetic field is uniform; in our linac we have field free regions between sections, which account for a $\simeq 20\%$ loss in total current;

d) the particle motion is considered extremely relativistic; J. Haimson has shown that there are bunching effects in the first section after the converter;

FIG. 1 - Storage ring lay-out.

e) only the positrons with energies between 7. 5 and 12. 5 MeV are considered; the actual magnetic collection system has a good acceptance also for particles with energies around 3, 3 MeV, 2 MeV and so on; however their phase spread, and consequently, the energy spread, should be considerably larger.

The experimental results obtained in Palo Alto at the end of the first high energy section gave a total current of 1. 9 mA; of the above men

Peak positron current per 1% energy bin
Electron beam on the converter: 250 mA at 80 MeV.
Total positron current between 291 and 333 MeV: 0.93 mA
Pulse length: 3.2 μsec; rep. rate: $50/_{16}$ p.p.s.
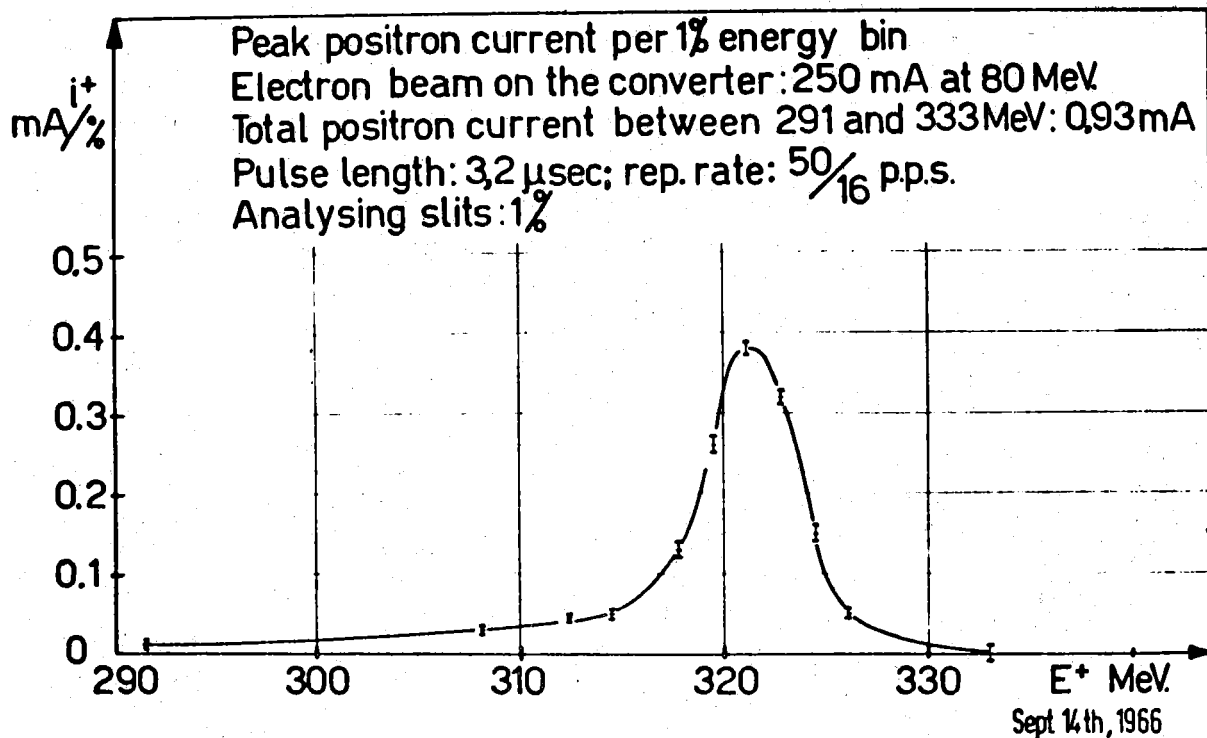Analysing slits: 1%

Sept 14th, 1966

FIG. 3 - Positron current spectrum.

tioned corrections, only c) should be applied to this result, to compare it with the 0.93 mA obtained at the end of the linac.

While the value of the correction a) might be questionable, it is sure that both the experimental results give a current higher than expected, taking into account the fact that further optimization might increase the positron current. The reason seems to be point e) above; its contribution is difficult to calculate, and more so if one wants to include bunching effects (point d)).

As a conclusion we can say that while the total positron current obtained is higher than the expected value, it could be still increased, according to the results at the end of the first waveguide; it is possible however that this increase will be in energy regions far from the peak. As far as the emittance is concerned, it has not been measured, but the axial magnetic field and the iris aperture limit it to π mrad cm at 360 MeV, which is the value accepted in the ring.

The energy acceptance of the ring is 2%; with the 0.58 mA of positrons, and assuming a factor of two loss in injection optics, the time required to get 100 mA circulating beam should be about 4 minutes[8].

It is worth mentioning the solution we adopted for the correction of solenoid misalignements. If we assume a rigid, ideal solenoid, tilted as compared to the beam axis, four independent steerings are needed for correcting the tilt, two for each plane. We used iron plates at the ends of each solenoid, precisely centered on the linac axis; they tend to give
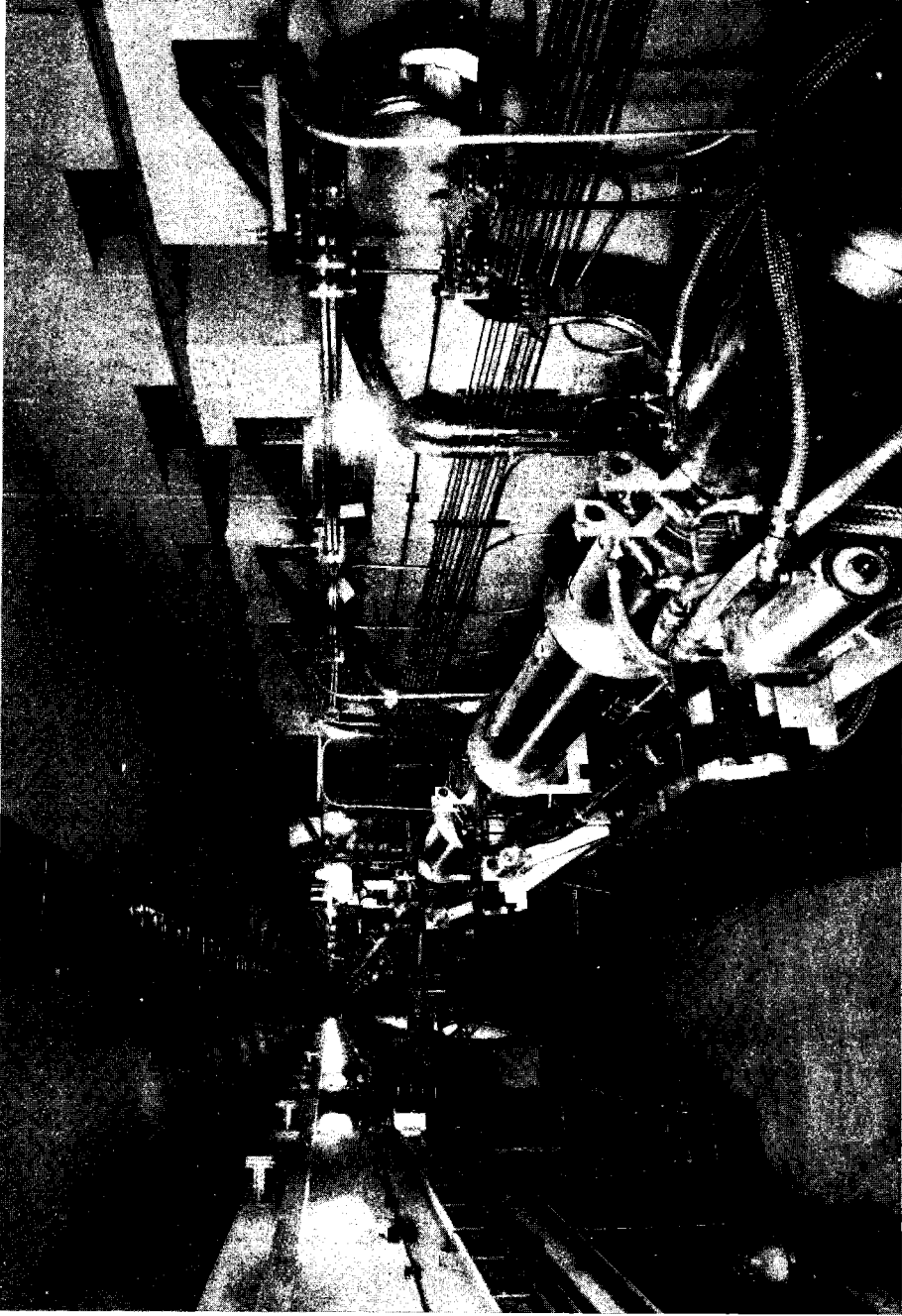
FIG. 2 – The linear accelerator seen from the injector.

centered fringings, so that two steerings only per solenoid are needed; the system seems to work well and the misalignement correction took not more than a couple of hours for the eight solenoids.

Injection optics and pulsed inflector. -

The transport channel for e^+ and e^- is composed of one pulsed magnet, five bending magnets and 23 quadrupoles; they are being assembled now and magnetic measurements will be done before installation.

The pulsed inflector (see fig. 5) has been satisfactorily tested at full power under vacuum; 100.000 pulses have been done without any mechanical adjustment of the spark-gaps (every filling requires $\sim$ 400 pulses).

Magnet and magnet power supply. -

Magnetic measurements have been made on quadrupole and magnet prototypes: a small change on the bending magnet pole profile has been made (the pole tips have been slightly increased in sise). All the rest is within tolerances.
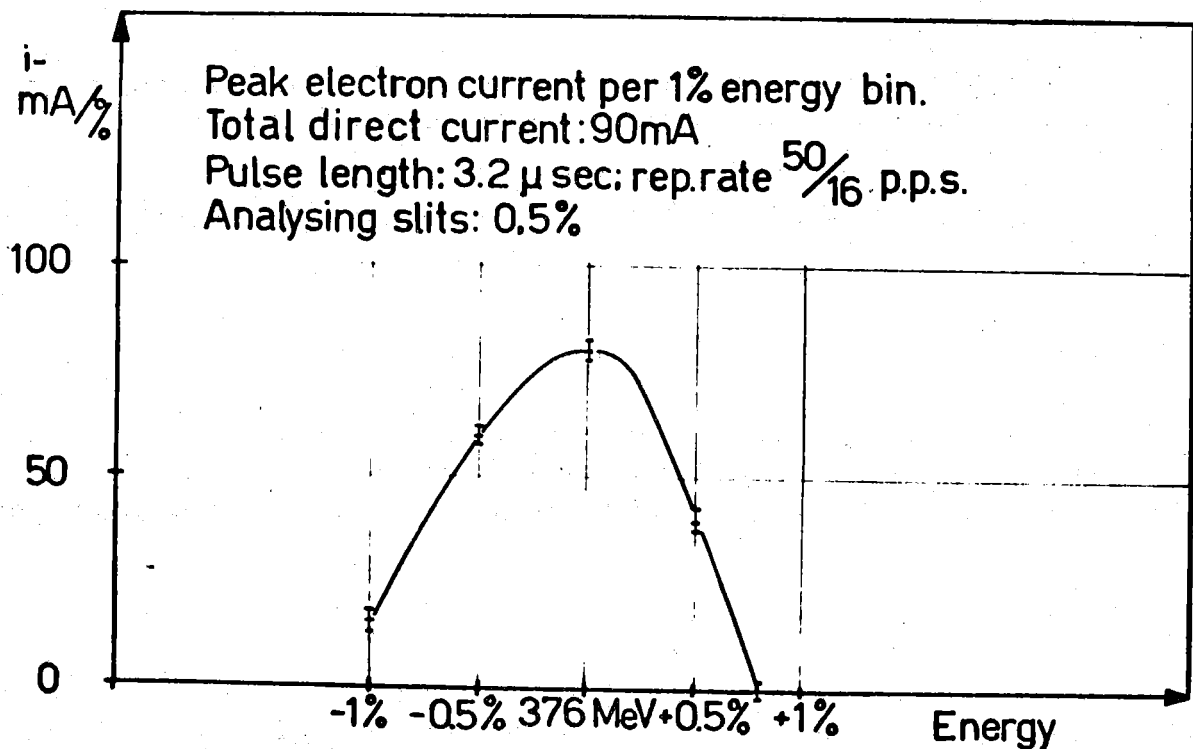
FIG. 4 - Electron current spectrum.

The production of quadrupoles started in July, 66, and the first 8, out of 48, should be shipped during next October; the magnet production started at the beginning of September.

The delivery should proceed at the rate of 10÷12 quadrupoles and 3 bending magnets per month, this means that all the magnetic structu re should be in Frascati by the middle of February 1967. The installation will start next November, with the first units.

The magnet power supply will be installed during October 66.

Vacuum system. -

The parts of the vacuum chamber are being delivered now (see fig. 6); the final cleaning process is being done in our Laboratory.

All the pumps have already been delivered.

RF system. -

The design of the RF cavity has been revised, because of mechanical problems due to the size (about 4 m in diameter). The two cavities are being built in Aluminum; the first one should be ready for electrical tests in January 67.

Full power tests made on the prototype cavity have been satisfactory: the phase control, which must keep the phase of the two pairs of cavities within ± 5°, operates well[9].

The power supply and amplifier are being installed in the final location.

Buildings. -

All the buildings have been completed: fig. 7 shows the storage ring hall (in July 66): the control and counting rooms are 3,2 meters higher than the hall floor; fig. 8 is an aerial view of the site (September 1966).

Time schedule. -

At the present time the part that is holding up the ring operation is the magnet; if the delivery of the last units will be in February 67, the installation will be completed by middle of March and we should have the first beam into the ring before the end of April 67.

The first beam tests will be done without any baking of the vacuum chamber, and, probably, the gas pressure will be in the high 10^{-8} torr range in presence of the beams.
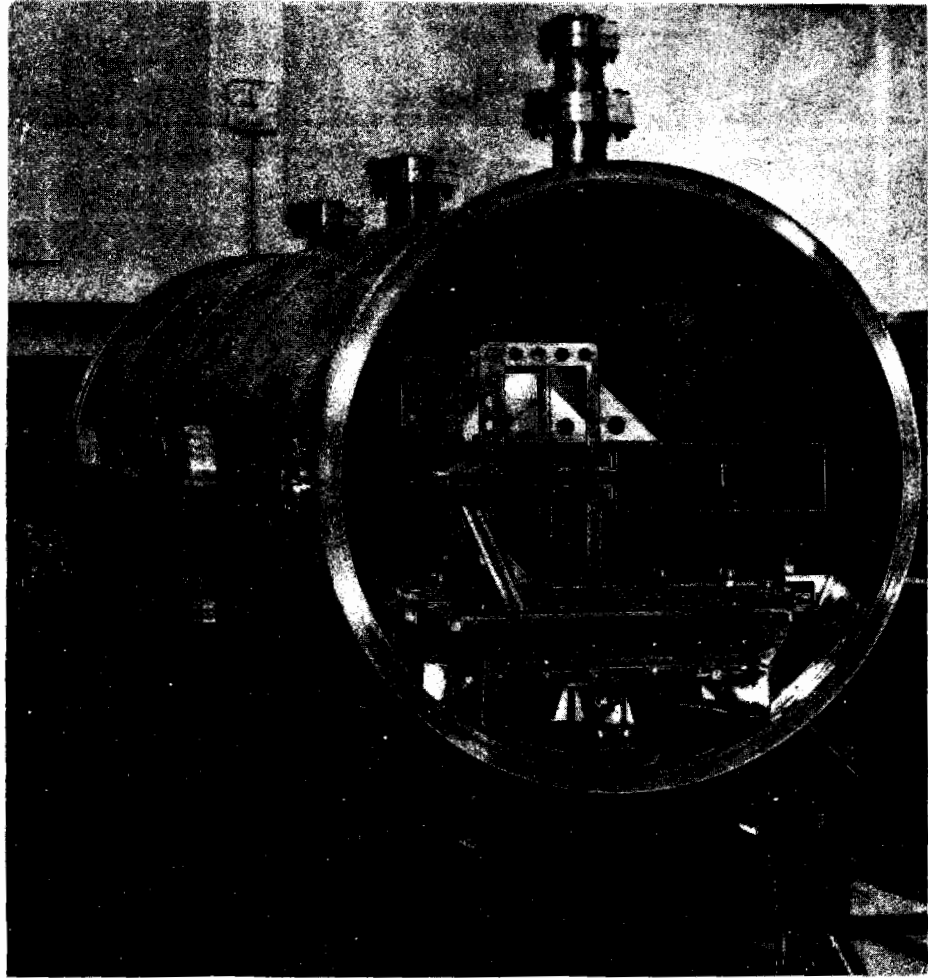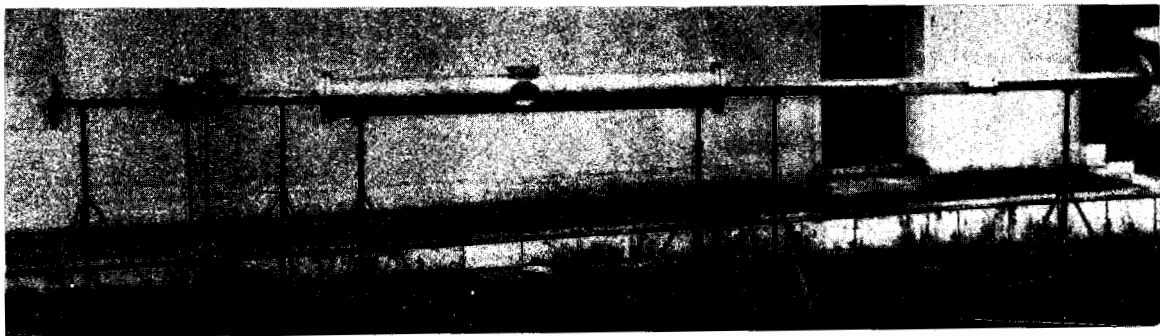
FIG. 5 - The pulsed inflector: beam input side.

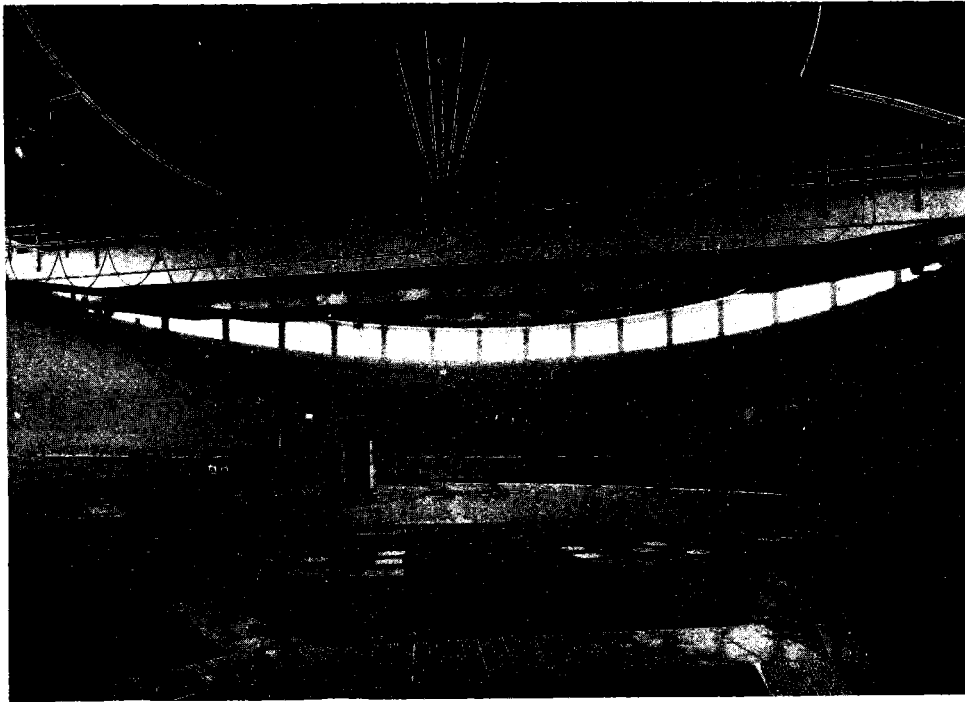FIG. 6 - The vacuum chamber - 1/12 of the circumference

FIG. 7 - The storage ring hall (July 1966).

FIG. 8 - Aerial view of the Frascati Laboratory (Sept. 1966).

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2) - INSTABILITY OF INTENSE ELECTRON BEAM AND RADIOFRE-QUENCY SYSTEM IN A STORAGE RING.

M. Bassetti

In this paper we will treat a problem that has been already investigated theoretically by many authors[1, 2, 3]. In this new work we make use of the method of finite differences that gives a simple interpretation of the physical phenomena.

Weak beam loading. -

In such a case the beam does not influence the cavity. We will consider separately the cavity and the beam synchrotron oscillations.

a) Treatment of the cavity.

We will consider schematically the cavity as a RLC shunt circuit. We will associate to the electromagnetic field of the cavity the vector

$$\vec{v}(t) = \begin{pmatrix} v(t) \\ i(t) \end{pmatrix}$$

where $v(t)$ is the voltage on the capacitance and $i(t)$ is the current in the inductance. The vector $\vec{v}(t)$, under the periodic impulse of the amplifier, will tend asinptotichally to a periodic vector $\vec{v}_c(t)$.

Generally speaking we may write

(1) $$\vec{v}(t) = \vec{v}_c(t) + \vec{v}_r(t).$$

One may show that the time dependance of $v_r(t)$ can be written as

(2) $$\vec{v}_r(t + \Delta t) = e^{(-\alpha I + \beta J) \Delta t} \vec{v}_r(t)$$

where

$$I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\alpha = 1/(2RC)$$

$$\beta^2 = (1/LC) - \alpha^2$$

$$J = \begin{vmatrix} -\alpha/\beta & -1/\beta\ C \\ 1/\beta\ L & \alpha/\beta \end{vmatrix}.$$

Let us note that accordingly to what happens in the treatment of the betatron oscillations it results:

(3) $\qquad\qquad e^{\beta Jt} = I \cos\beta\ t + J \sin\beta\ t.$

Let us further assume

$$\beta\ T = 2\pi + \theta$$

(4) $\qquad\qquad \vec{v}_{rn} = \vec{v}_r(t_p)$

$$\vec{v}_{r(n+1)} = \vec{v}_r(t_p + T)$$

where we indicate with t_p, $t_p + T$, $t_p + 2T$, ... the transit times of the synchronous particle.

From (2) using the formula (3) and (4) we deduce

(5) $\qquad\qquad \vec{v}_{r(n+1)} = e^{-\alpha T}\ e^{J\theta}\ \vec{v}_{rn}.$

Solving the vector equation (5) it results that each of the two components of $\vec{v}_{rn}$ is a linear combination of two solutions of the type x^n with:

(6) $\qquad\qquad x = e^{-\alpha T \pm i\theta}$

b) Equations of small synchrotron oscillations.

Making use of finite difference for small synchrotron oscillations of the beam we have:

(7) $\quad \left[\tau(n+2) - \tau_s\right] - 2 e^{-\gamma} \cos\delta \left[\tau(n+1) - \tau_s\right] + e^{-2\gamma}\left[\tau(n) - \tau_s\right] = hv_{r(n+1)}$

where with $\tau(n)$ we indicate the beam delay at the nth turn with respect to the nth supply impulse and with τ_s the synchronous one. h is a constant and $v_{r(n+1)}$ is the first component of $\vec{v}_{r(n+1)}$.

The solution of (7) is a linear combination of four solutions of the type x^n with

(8) $\qquad\qquad x = e^{-\gamma \pm i\delta} \qquad$ or $\qquad x = e^{-\alpha T \pm i\theta}.$

γ and δ are the values of the damping and of the angular frequency of the synchrotron oscillations computed for a turn.

Strong beam loading. -

While for weak beam intensity $\vec{v}_{rn}$ evolves independently from the beam according to (5), when the beam intensity is not negligible the cavity is subjected also to the impulses of the beam which are not periodic if the beam is not synchronous.

In this case $\vec{v}_{rn}$ cannot be zero and we must define it again.

After having linearized the equations, the coupling is taken mathematically into account by introducing into (5) a new term proportional to $\left[\tau (n+1) - \tau (n) \right]$ and to the beam loading.

If we consider equation (7) and the two scalar equations of the modified (5) we have a system of 3 finite differences equations. From the evaluation of the solution of this system arises a polynomial of 4th degree of the type:

$$(9) \qquad a_o x^4 + (a_1 + \Delta a_1) x^3 + (a_2 + \Delta a_2) x^2 + (a_3 + \Delta a_3) x + a_4 = 0$$

where Δa_1, Δa_2, Δa_3 are proportional to the beam intensity.

From (9) it follows that a_o and a_4 do not depend on beam intensity and therefore the product of the four solutions is constant and it is given by $e^{-2(\alpha T + \gamma)}$.

The stability condition imposes that the four solutions of (9) have each an absolute value smaller than 1. In accord with ref. (1,2,3) (with the hypothesis of a cavity schematicable with an RLC circuit) also in this case it follows that, neglecting the damping we have stability only for a negative θ, that is when the driving frequency is greater than the resonant frequency of the cavity.

At the present time we are working on the following points:

a) the study of the limits for stability as a function of θ and of beam intensity (considering also the synchrotron damping);

b) the study of the coupling abandoning the hypothesis of a point-like beam and assuming a given structure of the beam. In such a case the impulse that the beam gives to the cavity can be expressed as an instantaneous perturbation on the voltage and in the current (and not only on the voltage); this point also will be studied from the point of view of the stability.

14.

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3) - INCREASE OF THE LONGITUDINAL DIMENSION OF A RELATIVISTIC BUNCH DUE TO THE COHERENT-RADIATION FORCE.

E. Ferlenghi

We shall denote $\eta = \psi - \psi_s$ the phase difference (referred to the radiofrequency, RF) between a generic particle and a synchronous one.

If we neglect the damping due to incoherent radiation and introduce the force due to coherent radiation, then the equation of synchrotron motion is:

(1)
$$\ddot{\eta} = -\Omega^2 \eta + \frac{cq\alpha\omega_s}{E_s}(F - F_s)$$

where

(2)
$$\Omega^2 = \frac{(eV \sin \psi_s)q\alpha\omega_s^2}{2\pi E_s}$$

F, F_s are the coherent radiation forces acting upon a generic particle and a synchronous one respectively; they are negative quantities, because the forces act in the direction opposite to the motion of the bunch; $(eV \sin \psi_s)$ is the energy supplied by the RF to the synchronous particle; q is the harmonic number of the RF; α is the momentum compaction; ω_s is the revolution frequency; E_s is the synchronous energy.

For the force we assume the expression given by L. V. Iogansen and M. S. Rabinovich[1] which is based upon the following hypotheses: a) the electromagnetic fields are the Liénard-Wiechert's ones, i. e. there is no shielding; b) the bunch is extremely thin; c) the effective angular dimension η_o of the bunch and the energy of the particles are such as to satisfy the relation $\gamma^{-3} \ll \eta_o \ll 1$.

Hence the force depends on the particle angular position referred to a certain origin (see formula (14) in (1)).

If we take as origin the phase ψ_s of the synchronous particle (obviously doing that we neglect possible effects of the coherent radiation on the motion of the origin itself), the force can be written in the form:

(3)
$$F = -\frac{2}{3^{1/3}} \frac{Ne^2}{R^2} q^{4/3} \frac{d}{d\eta} \int_0^\infty \psi(\eta - \zeta)\zeta^{-1/3} d\zeta .$$

where: N is the number of particles in the bunch; e is the electron charge; R is the machine radius; $\psi(\zeta)$ is the phase distribution function.

The force acting upon the synchronous particle will be:

16.

(4)
$$F_s = F\big|_{\eta = 0} \; .$$

Let us introduce the distribution function $f(\eta, \dot{\eta}, t)$ obeying to the equation:

$$\frac{\partial f}{\partial t} + \dot{\eta} \frac{\partial f}{\partial \eta} + \ddot{\eta} \frac{\partial f}{\partial \dot{\eta}} = 0$$

where $\ddot{\eta}$ is to be substituted by the expression (1). We seek a stationary solution ($\partial f / \partial t = 0$) of this equation, with:

$$f(\eta, \dot{\eta}) = \mathcal{P}(\eta) w(\dot{\eta}) \; .$$

The "momenta" distribution function is unaffected by the new terms of force (3) and (4).

For the phase distribution function we obtain the equation:

(6)
$$\ln \mathcal{P} = -a^2 \Omega^2 \left\{ \frac{\eta^2}{2} + \frac{\lambda^2}{\Omega^2} \int_0^\infty \mathcal{P}(\eta - \xi) \xi^{-1/3} d\xi - \right.$$
$$\left. - \frac{\lambda^2}{\Omega^2} \left[\frac{d}{d\eta} \int_0^\infty \mathcal{P}(\eta - \xi) \xi^{-1/3} d\xi \right]_{\eta = 0} \right\} + C \; ,$$

with a^2, separation constant and

(7)
$$\lambda^2 = \frac{c q \varkappa \omega_s}{E_s} \frac{2}{3^{1/3}} \frac{N e^2}{R^2} q^{4/3} \; .$$

We can derive an approximated solution of the eq. (6) in the hypothesis that the perturbation method holds.

Then the zero order solution is:

(8)
$$\mathcal{P}_0 = \overline{\mathcal{P}}_0 \exp\left(-\frac{1}{2} a^2 \Omega^2 \eta^2 \right).$$

After defining the mean square deviation of η, η_0 and by taking account of the normalization condition for $\mathcal{P}_0$, we obtain:

(9)
$$\mathcal{P}_0 = \frac{1}{\sqrt{2\pi} \, \eta_0} \exp\left(-\frac{1}{2} \frac{\eta^2}{\eta_0^2} \right).$$

By substituting (9) into the right hand side of the eq. (6), we derive the first order solution $\mathcal{P}_1$.

The result shows that for big ($|\eta| \geq 4.5 \, \eta_0$) and small ($|\eta| \leq 0.45 \, \eta_0$) values of η the function $\mathcal{P}_1$ is well approximated by a gaus

sian function and for intermediate values the deviations of $\mathcal{S}_1$ from a gaussian are small.

Then we assume for evaluating purposes that again $\mathcal{S}_1$ is a gaussian distribution:

(10)
$$\mathcal{S}_1 \sim \exp\left\{-\frac{\eta^2}{2\eta_o^2}\left[1 - \frac{\lambda^2}{\Omega^2}\frac{2^{-5/6}}{3}\frac{\Gamma(2/3)}{\Gamma(5/6)}\eta_o^{-7/3}\right]\right\} .$$

From this expression one can conclude that the coherent radiation causes an increase of the longitudinal dimension of the bunch.

Obviously the formula (10) is applicable if the perturbation method is consistent. Then the second term in brackets is to be a small perturbation term (say, $\leq 1/10$).

The applicability of (10) to the case of real accelerators is limited by the fact that the actual shielding existing in the accelerators reduces (as demonstrated by J. S. Nodvick and D. S. Saxon[2]) the power irradiated by a bunch of particles. Hence there is also a decrease of the force acting upon the bunch.

Therefore the results, given by the eq. (10) represent an upper limit for the coherent radiation effect on the phase distribution.

Let us assume that in the case of finite shielding the coherent radiation force decreases with respect to the no shielding case in the same way as does the radiated power in the corresponding situations. Therefore we admit that there are no appreciable variations in the character of the force, in other words no spectral distorsions.

Then if we employ in calculations the formulae for the radiated power referring to a uniform distribution (see the expressions (23), (24), (26) in (2)) and express the radiated power P_{coh} (finite shielding) in terms of $P_{coh}^{(o)}$ (no shielding), we find the reduction factor:

(11)
$$f = (6\,\eta_o/q)^{-2/3}\left\{\frac{\sqrt{3}\,a}{2R} + \frac{32}{\pi}\left[e^{-2\pi\delta_1} + e^{-2\pi\delta_2}\right]S(\frac{2\pi R\eta_o}{qa})\right\}$$

where the new symbols are:
a, the distance separating the shielding plates;

$$\delta_1 = \frac{R - R_1}{a} \quad ; \qquad \delta_2 = \frac{R_2 - R}{a} \quad ;$$

with R_1, inner radius and R_2 outer radius of the plates $(R_1 < R < R_2)$;

$$S(y) = \frac{1}{2}\left[C + \ln y - Ci(y)\right]$$

with C, Euler's constant and Ci(y) the cosine integral.

The factor f is defined by equality:

$$P_{coh} = f\, P_{coh}^{(o)}$$

(For example, we obtain for the storage ring Adone, with $\eta_o = 5 \cdot 10^{-2}$, a reduction factor $f \approx 1/3$).

In conclusion the condition of applicability of (10) is:

$$(12) \qquad \eta_o^{7/3} \gg f q^{4/3}\, \frac{Ne^2}{R(eV \sin \psi_s)}$$

It must be stressed again that for the validity of the perturbation method the coherent radiation effect in any case must be smaller than $\sim 10\%$. In spite of this limitation the interest of the calculation is based essentially on two considerations:

a) - the values of η_o satisfying the condition (12) lie in the operating region of the actual accelerators. For example in the case of Adone it must be $\sim \eta_o \gtrsim 5 \cdot 10^{-2}$ (or the lenght of the bunch larger than ~ 50 cm);

b) - with smaller values of η_o, when the condition (12) is no more satisfied (as in Adone in an intermediate energy region), the perturbation method is not valid, however the equilibrium dimension of the bunch cannot exceed, as order of magnitude, the one allowed by our approximation (10).

For further details see reference (3).

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4) - REMARKS ON MULTIPLE COULOMB SCATTERING IN A RELATIVISTIC ELECTRON BEAM, BEAM SIZE AND LIFETIME.

C. Pellegrini

It is well known that the Coulomb scattering of two electrons belonging to the same bunch of a relativistic beam moving inside a circular accelerator, changes the particle energy and hence influences both the lifetime[1] and the beam size[2]. Up to now this effect has been evaluated in the case of a flat beam, i. e. a beam whose dimension in the direction orthogonal to the medium plane of the machine is zero, and when the scattering is non relativistic in the center of mass system (C. M.) of the two electrons.

We will try in this note to clear up how good are these approximations.

We think that this could be of some importance, in view of the relevant part that the beam size and lifetime play in the operation of a storage ring.

Let us consider two electrons whose displacement with respect to a reference trajectory (R. T.) of arc lenght "s" is defined by[3]

$$\delta P_i = \underline{\alpha} \sigma_i(s) + \underline{\beta} x_i(s) + \underline{\gamma} z_i(s) \qquad i = 1,2$$

$\underline{\alpha}, \underline{\beta}, \underline{\gamma}$ being an orthonormal triad on R. T. with $\underline{\alpha}$ tangent to R. T. and $\underline{\beta}$ the principal normal.

The momentum of the two particles having energy E_i is given by

$$\underline{q_i} = E_s \left\{ (1 + p_i)\underline{\alpha} + (x_i' + K\sigma_i - Hz_i)\underline{\beta} + (z_i' + Hx_i)\underline{\gamma} \right\}$$

where E_s is the energy of a particle following R. T. , $p_i = E_i - E_s/E_s$, $K(s)$, $H(s)$ are respectively the curvature and torsion of R. T. and $x' = dx/ds$. $z' = dz/ds$.

Then in a scattering event defined, in C. M. , by the angles χ, ψ, the energy variation $\varepsilon_i = \Delta E_i/E_i$ of the particles in the laboratory system is given, to first order in σ, x, z, p, by

(1)
$$\varepsilon_i = \pm \frac{\gamma \theta \, \text{sen} \, \chi \, \text{sen} \, \psi}{2\sqrt{1 + \gamma^2(\theta^2/4)}},$$

where

$$\gamma = \frac{E_s}{m_o c^2} \qquad \text{and}$$

(2)
$$\theta = \sqrt{(x_1' - x_2')^2 + (z_1' - z_2')}$$

     The function $P(\mathcal{E}/\theta)d\mathcal{E}d\theta$, giving the number of scattering events per unit time producing an energy variation $\mathcal{E}$ contained in the interval $\mathcal{E} \longmapsto \mathcal{E} + d\mathcal{E}$, when θ is in the interval $\theta \longmapsto \theta + d\theta$, can be evaluated using the scattering cross section and the θ distribution function $P(\theta)$. This last function can be obtained from the distribution function of x', z' which we assume to be[4]

(3)
$$P(x', z') = \frac{1}{\pi} \sqrt{\nu_{11}\nu_{22} - \nu_{12}^2} \ \exp -$$

$$- \left\{ \nu_{11}\left(\frac{dx}{ds}\right)^2 + 2\nu_{12}\frac{dx}{ds}\frac{dz}{ds} + \nu_{22}\left(\frac{dz}{ds}\right)^2 \right\}$$

Then from (2), (3) one has

(4)
$$P(\theta) = \frac{1}{2}\sqrt{\nu_{11}\nu_{22} - \nu_{12}^2} \ \theta \ e^{-\frac{\theta^2}{4}(\nu_{11} + \nu_{22})}$$

$$\cdot \ I_0\left[\sqrt{(\nu_{11} - \nu_{22})^2 + 4\nu_{12}} \ \frac{\theta^2}{4} \right] .$$

     In the limit $\nu_{22} \longrightarrow \infty$ (4) reduces to the one dimensional distribution

(5)
$$P(\theta) = \sqrt{\frac{\nu_{11}}{2\pi}} \ e^{-\theta^2 \nu_{11}/2} \ \triangleq \ \frac{1}{\sqrt{4\pi\langle x'^2\rangle}} \ e^{-\frac{\theta^2}{4\langle x'^2\rangle}}$$

used by the other authors[1,2].

     Introducing the variable

$$\zeta = \frac{\theta}{2}\sqrt{\nu_{11} + \nu_{22}}$$

one has also

(6)
$$P(\zeta) = \frac{\zeta}{\sqrt{K}} \ e^{-\zeta^2} I_0\left(\sqrt{1 - \frac{1}{K}} \ \zeta^2\right)$$

where K can be expressed as a function of the mean square value of x', z'; namely

$$K = \frac{1}{4}\frac{\left[\langle x'^2\rangle + \langle z'^2\rangle\right]^2}{\langle x'^2\rangle\langle z'^2\rangle - \langle x'z'\rangle^2} \ > \ 1$$

and in the limit of small coupling, $\langle x'^2 \rangle \gg \langle z'^2 \rangle$,

$$K \simeq \frac{1}{4} \frac{\langle x'^2 \rangle}{\langle z'^2 \rangle} .$$

The distribution function $P(\zeta)$ has a maximum for $\zeta \simeq 1$ and drops rapidly to zero for $\zeta > \sqrt{K}$.

With the help of (6) one can evaluate the quantities

(7)
$$\Lambda_n(\mathcal{E}_m, \mathcal{E}_M) = \int_{\mathcal{E}_m}^{\mathcal{E}_M} \mathcal{E}^n P(\mathcal{E}) d\mathcal{E} =$$

$$= \int_{\zeta_m}^{\zeta_M} d\zeta \int_{\mathcal{E}_m}^{B(\zeta)} \mathcal{E}^n P(\mathcal{E}/\zeta) d\mathcal{E} +$$

$$+ \int_{\zeta_M}^{\infty} d\zeta \int_{\mathcal{E}_m}^{\mathcal{E}_M} d\mathcal{E} \; \mathcal{E}^n P(\mathcal{E}/\zeta)$$

where

$$B(\zeta) = \frac{\gamma}{\sqrt{1 + \gamma^2 \dfrac{\zeta^2}{\nu_{11} + \nu_{22}}}} \; \frac{\zeta}{\sqrt{\nu_{11} + \nu_{22}}}$$

and $\zeta_{m(M)}$ is defined by

$$B(\zeta_{m(M)}) = \mathcal{E}_{m(M)}.$$

To determine the beam size we need the quantity

$$\langle \mathcal{E}^2 \rangle = \Lambda_2(\mathcal{E}_m, \mathcal{E}_{RF})$$

where $\mathcal{E}_{RF}$ is the radio-frequency acceptance and $\mathcal{E}_m$ is the low energy cutoff. Using the uncertainty principle[5] $\mathcal{E}_m$ can be defined as $\mathcal{E}_m = \hbar/d_m$, where $d_m = (1/2)(V/N)^{1/3}$ is the average half distance between the particles, V is the bunch volume, N the number of particles in the bunch.

Then, introducing the quantity

(8)
$$\zeta_{m(M)} = \frac{\mathcal{E}_{m(RF)}}{\gamma E_s} \sqrt{\nu_{11} + \nu_{22}}$$

one has

$$(9) \qquad \langle \varepsilon^2 \rangle = \frac{\sqrt{2}\, \pi\, r_0\, Nc}{\gamma^3\, v\, \sqrt{\langle x_i'^2 \rangle + \langle z_i'^2 \rangle}}\; \zeta_m\, F(\zeta_m, \zeta_M)$$

where

$$F(\zeta_m, \zeta_M) = \zeta_m \sqrt{K} \left\{ \int_{\zeta_m}^{\zeta_M} d\zeta\, \frac{P(\zeta)}{\zeta^3} \left[\frac{\zeta^2}{\zeta_m^2} \left(\ln \frac{\zeta}{\zeta_m} \right. \right. \right.$$

$$\left. \left. - \frac{1}{4} \right) + \frac{1}{4} \right] + \int_{\zeta_M}^{\infty} d\zeta\, \frac{P(\zeta)}{\zeta^3} \left[\frac{\zeta^2}{\zeta_m^2} \ln \frac{\zeta_M}{\zeta_m} \right.$$

$$\left. \left. - \frac{1}{4}\, \frac{\zeta_M^2}{\zeta_m^2} + \frac{1}{4} \right] \right\}$$

It is now possible to evaluate the root mean square value σ_p of the quantity $(E_i - E_s)/E_s$, by taking into account the effect of synchrotron radiation and of the multiple coulomb scattering.

The necessary numerical calculations, which are being made on a computer, by M. Bassetti and M. Buonanni, are not still finished.

However, preliminary results seem to shaw that the use of the distribution (6) instead of (5), gives a different dependence of σ_p on the machine energy, namely like $1/\sqrt{\gamma}$ instead of $1/\gamma$.

This work is part of a program of calculations on beam properties being performed by the Adone group.

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(5) - For a discussion on this point see reference (2).